

# Quantum decoherence in noninertial frames

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Quantum decoherence, which appears when a system interacts with its environment in an irreversible way, plays a fundamental role in the description of quantum-to-classical transitions and has been successfully applied in some important experiments. Here, we study the decoherence in noninertial frames for the first time. It is shown that the decoherence and loss of the entanglement generated by the Unruh effect will influence each other remarkably. It is interesting to note that in the case of the total system under decoherence, the sudden death of entanglement may appear for any acceleration. However, in the case of only Rob's qubit undergoing decoherence sudden death may only occur when the acceleration parameter is greater than a "critical point."

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## I. INTRODUCTION

The study of quantum information in noninertial framework is not only helpful for understanding some key questions in quantum mechanics [1–3], but it also plays an important role in the study of entropy and the information paradox of black holes [4, 5]. Recently, much attention has been focused on the topic of the quantum information in a relativistic setting [6–12] and, in particular, on how the Unruh effect changes the degree of quantum entanglement [13] and fidelity of teleportation [14]. However, it should be pointed out that all investigations in noninertial frames are confined to the studies of the quantum information in *an isolated system*. However, in a realistic quantum system, the *interaction* between the quantum system and the surrounding environment is inevitable, and then the dynamics of the system is non-unitary (although the combined system plus environment evolves in a unitary fashion). The decoherence [15, 16], which appears when a system interacts with its environment in an irreversible way, can be viewed as the transfer of information from system into the environment. It plays a fundamental role in the description of the quantum-to-classical transition [17, 18] and has been successfully applied in the cavity QED [19] and ion trap experiments [20].

In this article we investigate the quantum decoherence of Dirac fields in a noninertial system. For the sake of brevity

and without loss of generality, we consider only the amplitude damping channel [21], which is the most typical quantum noisy channel and can be modeled by the spontaneous decay of a two-level quantum state in an electromagnetic field [22]. We assume that two observers, Alice and Rob, share an entangled initial state at the same point in flat Minkowski spacetime. After that Alice stays stationary while Rob moves with uniform acceleration. We let one (or both) of the observers moves (or stays) in the noisy environment and discuss whether or not the quantum decoherence and the loss of entanglement generated by Unruh radiation will influence each other. A key question to be answered is: Does the entanglement appear to be sudden death [23] or does it only disappear as time tends to infinity?

We assume that Alice has a detector sensitive only to mode  $|n\rangle_A$  and Rob has a detector sensitive only to mode  $|n\rangle_R$ , and they share the maximally entangled initial state

$$|\Phi\rangle_{AR} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_R + |1\rangle_A|1\rangle_R), \quad (1)$$

at the same point in Minkowski spacetime, where  $\{|n\rangle_A\}$  and  $\{|n\rangle_R\}$  indicate Minkowski modes described by Alice and Rob, respectively. We then let Alice remain stationary while Rob moves with uniform acceleration. From the perspective of Rob the Minkowski vacuum is found to be a two-mode squeezed state [8]

$$|0\rangle_M = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II}, \quad (2)$$

where  $\cos r = (e^{-2\pi\omega c/a} + 1)^{-1/2}$ ,  $a$  is Rob's acceleration,  $\omega$  is

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frequency of the Dirac particle,  $c$  is the speed of light in vacuum, and  $\{|n\rangle_I\}$  and  $\{|n\rangle_{II}\}$  indicate Rindler modes in Region  $I$  and  $II$  (see Fig. 1), respectively. The only excited state is given by

$$|1\rangle_M = |1\rangle_I |0\rangle_{II}. \quad (3)$$

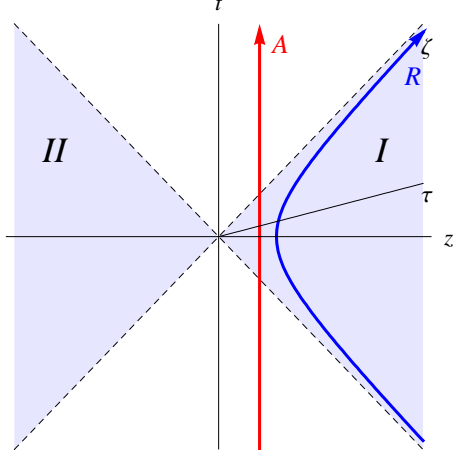


FIG. 1: (Color online) Rindler spacetime diagram: An accelerated observer Rob travels on a hyperbola in region  $I$  with uniform acceleration  $a$  and is causally disconnected from region  $II$ .

Using Eqs. (2) and (3), we can rewrite Eq. (1) in terms of Minkowski modes for Alice and Rindler modes for Rob

$$|\Phi\rangle_{A,I,II} = \frac{1}{\sqrt{2}} \left( \cos r |0\rangle_A |0\rangle_I |0\rangle_{II} + \sin r |0\rangle_A |1\rangle_I |1\rangle_{II} + |1\rangle_A |1\rangle_I |0\rangle_{II} \right). \quad (4)$$

Since Rob is causally disconnected from region  $II$ , the physically accessible information is encoded in the mode  $A$  described by Alice and mode  $I$  described by Rob. Tracing over the state in region  $II$ , we obtain

$$\rho_{A,I} = \frac{1}{2} \left[ \cos^2 r |00\rangle\langle 00| + \cos r (|00\rangle\langle 11| + |11\rangle\langle 00|) + \sin^2 r |01\rangle\langle 01| + |11\rangle\langle 11| \right], \quad (5)$$

where  $|mn\rangle = |m\rangle_A |n\rangle_I$ .

## II. CASE OF SINGLE QUBIT UNDERGOING DECOHERENCE

*Single qubit under decoherence case:* Now we consider Rob's state coupled to a dissipative environment, which corresponds to the spontaneous decay of Rob's state because it interacts with an electromagnetic field environment [22]. This process may be described as [16]

$$|0\rangle_R |0\rangle_E \rightarrow |0\rangle_R |0\rangle_E, \quad (6)$$

$$|1\rangle_R |0\rangle_E \rightarrow \sqrt{1-P_R} |1\rangle_R |0\rangle_E + \sqrt{P_R} |0\rangle_R |1\rangle_E. \quad (7)$$

Eq. (6) indicates that the system has no decay and the environment is untouched. Eq. (7) shows that, if decay exists in the system, it can either remain there with probability  $(1-P_R)$ , or be transferred into the environment with probability  $P_R$ . Usually, the dynamic of an open quantum system is described by a reduced density operator which is obtained from the density operator of the total system by tracing over the degrees of freedom of the environment. By considering the environment as a third system, we can obtain a unified entanglement-only picture.

The dynamics described by Eqs. (6) and (7) for a single qubit also can be represented by the following Kraus operators [24, 25]

$$M_0^R = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-P_R} \end{pmatrix}, \quad M_1^R = \begin{pmatrix} 0 & \sqrt{P_R} \\ 0 & 0 \end{pmatrix}, \quad (8)$$

where  $P_R$  ( $0 \leq P_R \leq 1$ ) is a parameter relating only to time. Under the Markov approximation, the relationship between the parameter  $P_R$  and the time  $t$  is given by  $P_R = (1 - e^{-\Gamma t})$  [21, 22] where  $\Gamma$  is the decay rate.

As a first step toward the study of quantum decoherence, we rewrite the state Eq. (5) as

$$\rho_{A,I} = \frac{1}{2} \left[ |0\rangle_A \langle 0| \otimes T_R^{00} + |0\rangle_A \langle 1| \otimes T_R^{01} + |1\rangle_A \langle 0| \otimes T_R^{10} + |1\rangle_A \langle 1| \otimes T_R^{11} \right], \quad (9)$$

with

$$T_R^{00} = \begin{pmatrix} \cos^2 r & 0 \\ 0 & \sin^2 r \end{pmatrix}, \quad T_R^{01} = \begin{pmatrix} 0 & 0 \\ \cos r & 0 \end{pmatrix}, \\ T_R^{10} = \begin{pmatrix} 0 & \cos r \\ 0 & 0 \end{pmatrix}, \quad T_R^{11} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

This form of the state suggests a natural bipartite split. We can use it to study how the environment effects Rob's single qubit. Under the amplitude damping channel, the state evolves to

$$\rho_s = \frac{1}{2} \begin{pmatrix} 1 - \beta \sin^2 r & 0 & 0 & \sqrt{\beta} \cos r \\ 0 & \beta \sin^2 r & 0 & 0 \\ 0 & 0 & P_R & 0 \\ \sqrt{\beta} \cos r & 0 & 0 & \beta \end{pmatrix}, \quad (10)$$

where  $\beta = 1 - P_R$ .

It is well known that the degree of entanglement for two-qubits mixed state in noisy environments can be quantified conveniently by concurrence, which is defined as [26, 27]

$$C_s = \max \{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad \lambda_i \geq \lambda_{i+1} \geq 0, \quad (11)$$

where  $\sqrt{\lambda_i}$  are square root of the eigenvalues of the matrix  $\rho_s \tilde{\rho}_s$ , where  $\tilde{\rho}_s = (\sigma_y \otimes \sigma_y) \rho_s^* (\sigma_y \otimes \sigma_y)$  is the "spin-flip" matrix for the state (10) which is given by

$$\tilde{\rho}_s = \frac{1}{2} \begin{pmatrix} \beta & 0 & 0 & \sqrt{\beta} \cos r \\ 0 & P_R & 0 & 0 \\ 0 & 0 & \beta \sin^2 r & 0 \\ \sqrt{\beta} \cos r & 0 & 0 & 1 - \beta \sin^2 r \end{pmatrix}. \quad (12)$$

Hence, the eigenvalues of  $\rho_s \tilde{\rho}_s$  are

$$\begin{aligned}\lambda_1 &= \frac{\beta}{4} \left[ \cos^2 r + \left( \cos r + \sqrt{\cos^2 r + P_R \sin^2 r} \right)^2 \right], \\ \lambda_2 &= \frac{\beta}{4} \left[ \cos^2 r + \left( \cos r - \sqrt{\cos^2 r + P_R \sin^2 r} \right)^2 \right], \\ \lambda_3 &= \lambda_4 = \frac{\beta}{4} P_R \sin^2 r.\end{aligned}\quad (13)$$

By using Eq. (11) we get the concurrence which is  $\cos r$  when the decay parameter  $P_R = 0$ , in which case our result reverts to that of Ref. [8].

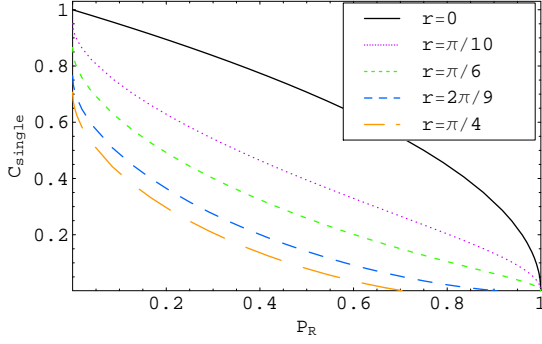


FIG. 2: (Color online) Concurrence as a functions of the decay parameter  $P_R$  with some fixed acceleration parameters [ $r = 0$  (black line),  $\frac{\pi}{10}$  (dotted line),  $\frac{\pi}{6}$  (dashed green line),  $\frac{2\pi}{9}$  (dashed blue line),  $\frac{\pi}{4}$  (dashed orange line)] when only Rob's qubit undergoes decoherence.

In Fig. (2) we plot the behavior of the concurrence which shows how the acceleration of Rob would change the properties of entanglement when his qubit couples to the environment. It is shown that, compared with the case of  $P_R = 0$  [8] (isolated system), the degree of entanglement decreases rapidly as acceleration increases. It is worth to note that Alsing *et al* [8] found that the entanglement of Dirac fields in an isolated system is not completely destroyed even in the limit case that Rob is under infinite acceleration. But we find that the entanglement of Dirac fields could tend to zero for finite acceleration. That is to say, the noise can greatly influence the loss of the entanglement generated by Unruh effect. Note that  $P_R$  is a monotonically increasing function of the time, this figure in fact describes the time evolution of entanglement of a bipartite system when one of them is coupled to an amplitude damping environment. It is interesting to note that the entanglement only disappears as  $t \rightarrow \infty$  when the acceleration is small or zero. However, the sudden death of entanglement appears at a finite time for large and infinite accelerations. Obviously, in the time evolution of entanglement there exists a “critical point” for the acceleration parameter. We note that the concurrence  $C_s = 0$  if the acceleration parameter  $r$  and the decay parameter  $P_R$  satisfy the relation

$$r = \arcsin \left( \frac{\sqrt{P_R^2 + 4} - P_R}{2} \right). \quad (14)$$

Considering the condition  $0 \leq P_R \leq 1$ , we find that sudden death of the entanglement will appear when  $\arcsin[(\sqrt{5} - 1)/2] \leq r \leq \frac{\pi}{4}$ . Thus, the “critical point” is  $r_c = \arcsin[(\sqrt{5} - 1)/2] = 0.666239$  below which sudden death of the entanglement can not take place.

### III. CASE OF TWO QUBITS UNDERGOING DECOHERENCE

*Two qubits under decoherence case:* Now we consider both Alice and Rob's states coupled to the noisy environment, which acts independently on both their states. The total evolution of this two qubits system can be expressed as

$$L(\rho_{AR}) = \sum_{\mu\nu} M_\mu^A \otimes M_\nu^R \rho_{AR} M_\nu^{R\dagger} \otimes M_\mu^{A\dagger}, \quad (15)$$

where  $M_\mu^i$  are the Kraus operators

$$M_0^i = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1 - P_i} \end{pmatrix}, \quad M_1^i = \begin{pmatrix} 0 & \sqrt{P_i} \\ 0 & 0 \end{pmatrix}, \quad (16)$$

where  $i = (A, R)$ ,  $P_A$  is the decay parameter in Alice's quantum channel and  $P_R$  is Rob's decay parameter. Here we only consider the global channels [21], in which all the subsystems are embedded in the same environment (i.e.,  $P_A = P_R = P$ ).

When both of the two qubits are coupled to the environment, state Eq. (5) evolves to

$$\rho_t = \frac{1}{2} \begin{pmatrix} 1 + P^2 - \tilde{\beta} \sin^2 r & 0 & 0 & \tilde{\beta} \cos r \\ 0 & \tilde{\beta}(P + \sin^2 r) & 0 & 0 \\ 0 & 0 & P\tilde{\beta} & 0 \\ \tilde{\beta} \cos r & 0 & 0 & \tilde{\beta}^2 \end{pmatrix}, \quad (17)$$

where  $\tilde{\beta} = 1 - P$ . We can easily get the “spin-flip” of this state and find that the matrix  $\rho_t \tilde{\rho}_t$  has eigenvalues

$$\begin{aligned}\tilde{\lambda}_1 &= \frac{\tilde{\beta}^2}{4} \left[ \cos^2 r + \left( \cos r + \sqrt{1 + P^2 - \tilde{\beta} \sin^2 r} \right)^2 \right], \\ \tilde{\lambda}_2 &= \frac{\tilde{\beta}^2}{4} \left[ \cos^2 r + \left( \cos r - \sqrt{1 + P^2 - \tilde{\beta} \sin^2 r} \right)^2 \right], \\ \tilde{\lambda}_3 &= \tilde{\lambda}_4 = \frac{\tilde{\beta}^2}{4} P(P + \sin^2 r).\end{aligned}\quad (18)$$

It is interesting to note that the concurrence is also  $\cos r$  for  $P = 0$ .

Figure (3) shows time evolution of quantum entanglement when the total two qubits system is coupled to the environment. It shows that, compared with the case of only Rob's qubit undergoing decoherence, the entanglement decreases more rapidly as the acceleration increases. It is interesting to note that the sudden death of entanglement appears at a finite time even for  $r = 0$ , and a larger acceleration also leads to an earlier appearance of the sudden death as the parameter  $P$  increases.

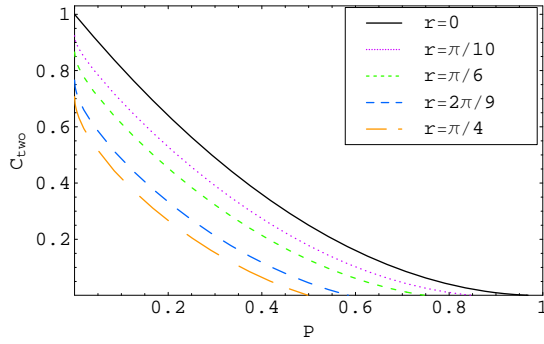


FIG. 3: (Color online) The concurrence as functions of the decay parameter  $P$  and acceleration parameter  $r$  [ $r = 0$  (black line),  $\frac{\pi}{10}$  (dotted line),  $\frac{\pi}{6}$  (dashed green line),  $\frac{2\pi}{9}$  (dashed blue line),  $\frac{\pi}{4}$  (dashed orange line)] when both Alice and Rob's qubits under decoherence.

In particular, when the acceleration approaches infinity, the sudden death appears when  $P \geq 1/2$ , whereas it happens when  $P_R \geq \sqrt{2}/2$  when only Rob's qubit undergoes decoherence. Thus, we come to the conclusion that the decoherence and loss of entanglement generated by the Unruh effect will influence each other in noninertial frames.

#### IV. SUMMARY

In conclusion, we have found that, unlike the isolated case in which the entanglement of Dirac fields survives even in the

limit of infinite acceleration [8], the entanglement could tend to zero for finite acceleration in this system; and a larger acceleration leads to an earlier disappearance of entanglement if either one or both subsystems experience a decoherence. Thus, the decoherence and loss of entanglement generated by the Unruh effect will influence each other remarkably in non-inertial frames. It is also shown that the sudden death of entanglement will appear for any acceleration when both of the two qubits interact with the environment. However, if only Rob's qubit undergoes decoherence, the sudden death only takes place when the acceleration parameter is greater than the "critical point",  $r_c = \arcsin[(\sqrt{5}-1)/2]$ . Our results can be applied to the case in which Alice moves along a geodesic while Rob hovers near the event horizon with a uniform acceleration and one or both of them are in an amplitude-damping environment.

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